An interpretation of Orbital Residuals of Earth Satellites as Evidence for Macroscopic Nonmetricity in Gravitation

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Abstract

The excess lunar secular acceleration $\dot{n}$, measured at $\dot{n}/n = (3.2 \pm 1.1) \times 10^{-11}/\mathrm{yr}$ by van Flandern is assumed to be due to a macroscopic nonmetricity field with an active Weyl vector which can be modelled by a scalar potential. The model predicts the known but, as yet, partially unexplained linear part of the orbital decay residuals of the LAGEOS satellite. The same model applied to the solar system predicts small effects for the planets and is therefore consistent with experimental limits on planetary ranging data. Unfortunately the orbital data for other earth satellites are not accurately enough known for comparison. However, we give predictions for the orbit of the Gravity Probe B (gyroscope) experiment which may be flown in a drag-free mode to the required accuracy.

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The most recent measurement of $\dot{n}$ for the moon obtained by van Flandern[1] is given in the form of the ratio (the logarithmic derivative of n)

$$\dot{n}/n = (3.2 \pm 1.1) \times 10^{-11}/\mathrm{yr} = (1.02 \pm 0.30) \times 10^{-18}/\mathrm{s} \qquad (1)$$

Although one is tempted to interpret this excess lunar secular acceleration $\dot{n}$ in terms of a time-varying gravitational constant G, the practice is generally model dependent.[1,2] If, for example, we use Kepler's Law, then $\dot{G}/G$ is just twice the value given by Eq. (1).[3] In this work, however, we will generally stay with the measured orbital elements and will show that this measured effect can be interpreted as a manifestation of a macroscopic, propagating nonmetricity field related to the non-zero covariant derivative of the metric for gravity, and that the basic structure of spacetime is that of a metric affine spacetime[4] with an active Weyl vector.[5]

Metric affine spacetimes may also contain torsion. But unlike nonmetricity, torsion has been investigated and incorporated into gravitational theories,[4] gauge theories of gravity,[6] and supergravity.[7] Gravitational theories with an active torsion vector have also been discussed in the context of a time varying gravitational coupling.[8,9] Most attempts have been directed at the construction of microscopic theories[4,5,10] and have concentrated on the spin properties of matter.[11] On those ocassions in which one has considered propagating torsion for macroscopic bodies, attention seemed to have been focused on the torsional effects on spinning matter such as tops.[12] One quickly discovers that, at best, the effects may show up at the post-post-

Newtonian level[13] which are presently far beyond the experimental capabilities of present day or proposed experiments on spin-precession such as for the orbiting ball gyroscope (Schiff) experiment.[14] One exception may be found in the work of Rauch[8] in which he attempts to compare the magnitude of the spin effects due to torsion to that of a time varying gravitational coupling. However, the model which is proposed contains only a torsion vector, and it has been shown that the torsion vector does not couple to the spin but is closely related to non-conservation of mass.[15] Instead here we direct our attention to the effect of nonmetricity, in particular the Weyl vector part, on the actual motion, i.e. to the orbital motion of the body itself. We mention that the treatment of orbital motion for theories with torsion has been considered.[16,17,18]

Typically one considers the motion of an orbiting and spinning test body from the standpoint of the equations of motion of the four momentum and the spin-tensor. These are essentially generalizations by Hehl[16] of the equations of motion due to Papapetrou.[19] They have also been further considered by Audretsch[17] and Hojman, Rosenbaum, and Ryan.[18] However, we are only interested in the orbital motion of the satellite; so that instead, we consider just the autotransport of the angular velocity vector, ω^j, of a satellite along the four velocity, $u^j = dx^j/ds$ in the frame of the central body

$$\nabla_u \omega^j = u^i \omega^j{}_{,i} + \Gamma_{ik}{}^j u^i \omega^k = 0, \tag{2}$$

where, for sake of completeness, $\Gamma_{ik}{}^j$ is the non-symmetric affine connection, latin indices run from 0, 1, 2, 3,

$$\Gamma_{ik}{}^{j} \equiv \{{}^{\,j}_{ik}\} + S_{ik}{}^{j} - S_{k}{}^{j}{}_{i} + S^{j}{}_{ik} + \tfrac{1}{2}(Q_{ik}{}^{j} + Q_{k}{}^{j}{}_{i} - Q^{j}{}_{ik}), \quad (3)$$

with $\{{}^{\,j}_{ik}\}$ the Christoffel symbol; $S_{ij}{}^{k}$ the torsion

$$S_{ik}{}^{j} \equiv \Gamma_{[ik]}{}^{j}, \quad (4)$$

defined by the antisymmetric part of the connection; and Q_{ijk} the nonmetricity

$$Q_{ijk} \equiv - \nabla_{i} g_{jk}, \quad (5)$$

is the non-zero covariant derivative of the metric with respect to the connection.[20] From this point on we will ignore the torsion terms. The prototype theory that we have in mind is a scalar-metric-nonmetric theory[21] which is very similar in construction (of the action) to the scalar-tensor-torsion theories of Rauch[8] and Germán.[9] In such a theory, the scalar field, ϕ, is directly related to the Weyl vector

$$Q_{i} = \tfrac{1}{4} Q_{ix}{}^{x} = \phi_{,i}/\phi \quad (6)$$

The nonmetricity is then given by

$$Q_{ij}{}^{k} = (\phi_{,i}/\phi)\ \delta_{j}^{k} \quad (7)$$

which shows that the Weyl vector is the only active component of the nonmetricity. This is, however, due to the simplicity of the model since no matter fields other than the scalar field are considered. [This is similar to the treatment of Rauch[8] and Germán[9].]

Consider now the low velocity limit of Eq.(2) for a flat background. In principle, one should not ignore the effect of the Christoffel part of the connection, but we are only looking for the unexplained excess in the motion of the moon. That is, the effect of the usual gravitational field is well understood and should be experimentally accounted for in the analysis of the data.[22] Any deviations then apparently lie outside these terms and is reported as an excess. In general, there will be corrections to Eq.(2) due to curvature just as there is in the equations of motion for the four momentum or the spin tensor.[15,16] However, for the simple model which we are considering here, we will neglect this complication.

In the low velocity limit, the angular velocity vector has only spatial components and the four velocity becomes $u^i = (1,0,0,0)$. Let the angular velocity be along the z-axis. Then Eq.(2) yields (no sum on z)

$$\dot{\omega}_z/\omega_z = - \Gamma_{0z}{}^z. \qquad (8)$$

In the flat background limit, we note that $\Gamma_{oz}{}^z = \frac{1}{2} Q_{oz}{}^z$ and the off-diagonal terms vanish.[23] Equation (8) then takes the form

$$\begin{aligned} \dot{\omega}_z/\omega_z &= -\tfrac{1}{2} Q_{oz}{}^z \\ &= -\tfrac{1}{2} Q_o \end{aligned} \qquad (9)$$

which is just the z-components of the ratio, $\dot{n}/n$, given by Eq. (1), i.e., the projection of n on to the axis of rotation of the earth (central body).

We now support our contention that $\dot{n}/n$ should be considered as evidence for macroscopic nonmetricity by resolving an unmodeled change in the semi-major axis of the orbit of the LAGEOS satellite.[24]

The LAGEOS satellite was launched in May, 1976, has a mass of 411 kg, optical corner reflectors, and an orbit with semimajor axis of 12,270 km, and an inclination of 109.9^0. It is completely passive and acts as a reference target for extraordinarily precise, ground based laser observations which are used to measure relative station positions on the earth, polar motion and the earth's gravitational constant. After all the known features of the earth's gravitational field are taken into account, Smith and Dunn[24] find a statistically significant, but unmodeled, uniform linear decrease in the semi-major axis of -1.1 mm/day. The spread in the data conservatively suggests an error less than 0.1 mm/day (our estimate). This effect persisted for the 32-month span from the date of launch for which the data were analyzed. Most recent data indicate the linear decrease in the semimajor axis has persisted to date.[25] In addition, analysis of the experimental data over a period of 6½ years shows evidence that the decrease could be as large as an average of -1.3 mm/day.[26] This decrease is equivalent to an excess secular acceleration of

$$(\dot{n}/n)_{LAGEOS} = (1.6 \pm 0.2) \times 10^{-15}/s \qquad (10)$$

Also the high altitude of the satellite puts it above the region of the atmosphere in which drag forces are known to affect satellite behaviour. There have been some attempts to explain the decrease in the semi-major axis (or the secular acceleration) of LAGEOS due to drag from neutral helium or charged particles.[27] Estimates[28] of about ten different effects seem to indicate that perhaps as much as 10% of the decrease could be accounted for by drag from neutral particles and spacecraft charging for 50-60% (if it occurred at a potential of -1V). An attempt to explain the decrease has been proposed that is based upon the effect of reflected radiation from an earth whose albedo exhibits hemispherical asymmetry.[29] A composite, Fourier analysis model which includes neutral and charged drag, plus a slightly different albedo model than above makes predictions on what the temperature profile and ion densities would have to be to explain the observed variations in the secular acceleration of LAGEOS.[30] This is probably the most sophisticated treatment of all effects to date and gives much of the general features of the data. However, actual experimental data over the orbit of LAGEOS for ion densities and temperatures are not very well known. These predictions do suffer, however, from the counter indication that the linear decrease has been uniform and has persisted in spite of the solar maximum which occurred after launch.[24] It has been suggested by Smith, et al.[26], after removal of the linear trend in the data, the residuals which seem to vary around zero may be due to the albedo model. These residual variations do seem to show a general rising trend during the first three years after launch; however, the maximum variation isn't reached until nearly 900 days after launch. However, they state that such a model is "...inadequate for explaining the observed structure in the along-track behavior of the LAGEOS orbit." We feel that the explanation for these residuals, after the linear part is removed, is most likely from drag forces caused by spacecraft charging. That the variations did not follow the profile of the exospheric temperature based on solar flux and geomagnetic activity, which peaked about 700 days after launch, does seems to indicate that drag forces (or reradiation forces) can not account for the entire variation and, most likely, not for the linear part of the orbital decay. This is, of course, the premise of our analysis.

We now propose that the two effects, the excess (unmodelled) secular acceleration of the moon and the linear orbital decay of the LAGEOS satellite are due to the same phenomenom. It would be very difficult to understand how the similar secular acceleration of the moon could be due to charge drag effects. The basis of our model is that the decrease for the moon, and at least a portion of the linear decrease in the orbit of LAGEOS, is due to a propagating macroscopic nonmetricity based upon a simple model in which the Weyl vector is obtainable from a scalar potential.

We propose a model for the scalar potential which is due to an interaction between the spin of the earth and the orbital angular momentum of the sattelite of the form

$$\Omega = \zeta|\vec{\omega}_E \cdot \vec{\omega}| \tag{11}$$

where $\vec{\omega}_E$ is the angular velocity of the earth, $\vec{\omega}$ is the orbital angular velocity of the satellite and ζ is an unknown coupling constant. The Weyl vector is then just the gradient of Ω

$$Q_j = \partial_j \Omega. \tag{12}$$

[There is some indication that consistent theories of gravitation can be obtained for metric affine spacetimes in which the Weyl vector is given by the gradient of a scalar. One only needs a slight modification of the usual gravitational scalar Lagrangian which takes into account the effects of volume preserving transformations of the connection.[21,31]] From the form Q_j and Eq. (9), it is easy to see that the variation in $\dot{n}/n$ will depend on the nonmetricity through the relation

$$Q_o = Q_{oz}{}^z = \frac{-3\zeta\, \omega_E\, (GM_E)^{\frac{1}{2}}\ \ \dot{r}\ |\cos\theta|}{2r^{5/2}} \tag{12}$$

where we have substituted the simple orbital relation for ω in order to demonstrate the dependence of Q_i on r. Since we are interested in long term effects, we must average the above quantities over an orbit. In this case, r is the semimajor axis, $\dot{r} \approx 4er/T$ is the mean radial motion (for one-half of a period), T is the orbital period, θ is the orbital inclination, and e is the eccentricity. We list the values of these parameters in Table 1 for various "satellites" as well as the data used in our calculations.

Table 1

Orbital Parameters

	Moon[32]	LAGEOS[24]	Mercury[32]	Venus[32]	Earth[32]	Sun[32]
semimajor axis (r)	3.84×10^{8}m	1.227×10^{7}m	5.795×10^{10}m	1.082×10^{11}m	1.496×10^{11}m	
mean radial motion ($\dot{r}$)	35.7 m/s	14.4 m/s	6270.8 m/s	152.2 m/s	317.1 m/s	
period (T)	2.36×10^{6}s	1.34×10^{4}s	7.6×10^{6}s	1.93×10^{7}s	3.156×10^{7}s	
eccentricity (e)	0.0549	0.003929	0.2056	0.006787	0.016722	
inclination (θ)	23°	110°	7°	3°	-	7°
$\dot{\omega}/\omega$ (input)	1.02×10^{-18}/s					
mass of central body (M)					5.98×10^{24}/kg	2×10^{30}kg
angular velocity of central body ω					7.27×10^{-5}/s	2.87×10^{-6}/s

Note that the inclinations of the Moon and LAGEOS are with respect to the equator of the earth; the others are with respect to the ecliptic.

Upon substituting Eq. (12) into Eq. (9), we obtain our model for the secular acceleration of a satellite

$$\frac{\dot{\omega}}{\omega} = -\tfrac{1}{2}\,\Omega_{,o} = \frac{3\zeta\;\omega_E(GM_E)^{\frac{1}{2}}\,\dot{r}\,|\cos\,\theta|}{4\;r^{5/2}} \tag{13}$$

Using the data given in Table 1, we find

$$(\dot{n}/n)_L \equiv (\dot{\omega}/\omega)_L = (\dot{\omega}/\omega)_M \, (r_M/r_L)^{2.5} \, (\dot{r}_L/\dot{r}_M) \, |\cos\theta_L/\cos\theta_M| \tag{14}$$

$$= 8.38 \times 10^{-16}/s$$

where the L and M stand for LAGEOS and the Moon, respectively, and have used the value for $\dot{n}/n$ for the moon as our input. The coupling constant is found to be

$$\zeta = 8.24 \times 10^{-2} \; s^2 \tag{15}$$

Note that the predicted value for $\dot{n}/n$ for LAGEOS is roughly 50% of the experimental value and, given the statistical variation of the data and the possibility of competing effects such as drag due to spacecraft charging, is well within the experimental range of the unaccounted portion of the secular acceleration in the angular velocity of LAGEOS.

From the form of Eq. (13), ζ will be a universal constant for any system of satellites, in particular the planets of the solar system. Table 2 gives the predicted values for the three inner most planets.

Table 2

Predicted values for logarithmic derivative of the secular acceleration for the inner planets

Planet	$\dot{n}/n$
Mercury	$1.59 \times 10^{-20}/s$
Venus	$8.06 \times 10^{-23}/s$
Earth	$7.50 \times 10^{-23}/s$

The increases predicted by the values of Table 2 are at most two orders of magnitude less than the experimental limits imposed by radar ranging to the planets.[3,33] Thus within the context of this model, one is able to reconcile the lunar data with the conflicting null results from solar system data. The predicted effect for the planets is too small to measure at present.

In conclusion, we make two final observations. The first is a prediction for the gravity probe B satellite. Ostensibly it is being flown to measure the predictions of Schiff[34] on the precession of an orbiting gyroscope - not for the decay of the orbit itself. Because of the required sensitivity of the experiment, the satellite will be flown in a drag compensating mode and with the addition of corner reflectors, the orbit could also be accurately determined. Although there are experimental reasons for a polar orbit, the gyroscope experiment will probably be inserted into orbit from the Shuttle Spacecraft and will be limited to an inclination of about 60° and a semimajor axis of about

7000 km. The orbit will probably be nearly circular; so we assume an eccentricity of about the same as LAGEOS of 0.004. This is then equivalent to assuming that the average radial velocity is about 19.2 m/s. One then easily predicts, using the same simplified model, an excess <u>decrease</u> in the semimajor axis of the order of 2.7 mm/day. Since the gyroscope experiment will be flown in a drag-free mode, the effect should be independent of drag providing the experimental limits on the drag-free mode can be made precise enough. As a result, an effect of this order of magnitude should be large enough to become evident in the planned year length flight of the experiment and therefore could provide an unambiguous test of the model.

We point out that the standard interpretation of the excess secular acceleration of the moon as a cosmological time-varying gravitational constant,[1,2,3,33,35] would, as we understand it, have no measurable effect on the LAGEOS or the Gravity Probe B satellites since presumably $\dot{G}/G$ would not vary spatially at the solar system scale as $\dot{n}/n$ does in this model. That means that a $\dot{G}/G$ variation would produce a secular acceleration [i.e., Eq. (1)] two or three orders of magnitude smaller for LAGEOS or Gravity Probe B than that predicted from propagating nonmetricity given by Eq. (13). Also the apparent inconsistency of the $\dot{G}/G$ interpretation for the moon[1,2] in comparison with the null results from radar ranging to the planets[3,33,35] implies a physically different mechanism. Eventually the radar-ranging experiments[3] may untangle the effect of $\dot{G}/G$ from that of a propagating nonmetricity. For this reason, it is important to note that should the radar ranging program eventually detect a non-zero value for $\dot{G}/G$ for Mercury, this value does not have to agree with that interpretation for the moon's data because of the different environment of the moon compared with the planets. This is tantamount to saying that G is also spatially varying. A different value of $\dot{n}/n$ from radar ranging would therefore tend to support a propagating nonmetricity field which is measurable at the solar system scale. We further speculate that the coupling constant, given by Eq. (15), could be a new universal constant. We will consider this aspect of our model in a future work.

Finally, that macroscopic gravity also contains a propagating nonmetricity field, evident and measureable at the solar system scale, may have far reaching consequences on our conception of the affect of nonmetricity on the galactic scale and even more so on the cosmological scale.[36]

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